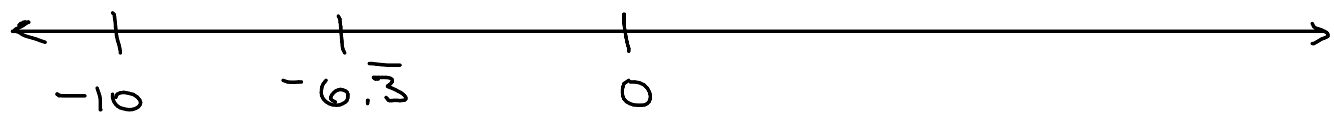
Unit 1 : Absolute Value and Radicals

1.1 : absolute value is the distance a number is from zero on a number line



$|-10| = 10$

$|-6.\overline{3}| = 6.\overline{3}$

$-|-7| = -7$

-7

p. 6

#1. k) $|-4| - |-2| = 2$

$4 - 2$

l) $|-4-(-2)| = 2$

m) $-|2| + |-2| = 0$

n) $|-2| + |-2| = 4$

#3. arrange from least → greatest

b) $\underbrace{-|2.\overline{5}|}_{-2.\overline{5}}$ $\quad \underbrace{|-2.\overline{5}|}_{2.\overline{5}}$ $\quad \underbrace{-|2.5|}_{-2.5}$ $\quad \underbrace{|-2.5|}_{2.5}$

1 4 2 3

c) $-|3-7|$, $|-(3-7)|$, $-|5-|-2||$, $-|5|-|-2|$

-4 $\quad$ 4 $\quad$ -3 $\quad$ -7

2 $\quad$ 4 $\quad$ 3 $\quad$ 1

remember... $\sqrt{20} = \sqrt{4}\cdot\sqrt{5}$
$= 2\sqrt{5}$

1.2 $\sqrt{\text{Radical}}$ Operations

$x^n = a$ $\qquad$ $x = \sqrt[n]{a}$ ← radicand

index

taking the "n^{th}" root

1) If a is positive and n is even ... 2 real n^{th} roots.

ex. $\sqrt{x^2} = \sqrt{81}$ $\qquad$ $x = \pm 9$

$9^2 = 81$

$(-9)^2 = 81$

$x^2 = 144$ $\qquad$ $x = \pm 12$

$x^4 = 16$ $\qquad$ $x = \pm 2$

2) If n is odd ... there is only 1 real n^{th} root

ex. $x^3 = 27$ $\qquad$ $x = 3$

$x^3 = -27$ $\qquad$ $x = -3$

3) If n is even, but a is negative ...

ex. $x^2 = -16$ no solution

Try:

p. 9

#2. k) $x^7 = 128$ $x = 2$

n) $x^9 = 0$ $x = 0$

j) $x^6 = -64$ $\varnothing$

i) $x^6 = 64$ $x = \pm 2$

Simplifying If x can be any real #, we may need $|\ |$ to ensure the result is a positive #.

ex. $\sqrt[2]{x^6} = |x^3|$ ← need $|\ |$ if variable has odd exponent

$x^3 \quad x^3$

$\sqrt{(x^3)^2}$

$\sqrt{x^8} = x^4$

$\sqrt{(x^4)^2}$

$\sqrt{x^2} = |x|$

$$\sqrt{x^5} = \sqrt{x^4}\cdot\sqrt{x}$$
$$= x^2\sqrt{x} \qquad x \geq 0$$

$$\sqrt{x^7} = \sqrt{x^6}\cdot\sqrt{x}$$
$$= |x^3|\sqrt{x} \qquad x \geq 0$$

$$\sqrt{x^{25}} = \sqrt{x^{24}}\cdot\sqrt{x}$$
$$= x^{12}\sqrt{x} \qquad x \geq 0$$

$$\sqrt[3]{x^{11}} = \sqrt[3]{x^9}\cdot\sqrt[3]{x^2}$$
$$= x^3\sqrt[3]{x^2}$$

$$\sqrt[4]{x^{19}} = \sqrt[4]{x^{16}}\cdot\sqrt[4]{x^3}$$
$$= x^4\sqrt[4]{x^3} \qquad x \geq 0$$

p. 10

\# 4.

a) $\sqrt{9x^2} = 3|x|$

c) $\sqrt{x^6y^4} = |x^3|y^2$

e) $\sqrt{x^2y} = |x|\sqrt{y} \qquad y \geq 0$

g) $\sqrt{x^2y^2} = |xy|$

i) $\sqrt{x^4y^2} = x^2|y|$

k) $\sqrt{x^5y^2} = x^2\sqrt{x}|y| \qquad x \geq 0$

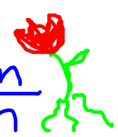
m) $\sqrt{16x^6y^8} = 4|x^3|y^4$

p. 11

#6. l) $\sqrt[3]{(x-1)\underbrace{(x^2-2x+1)}_{(x-1)(x-1)}} \rightarrow \sqrt[3]{(x-1)^3} = x-1$

p) $\sqrt[4]{\frac{81}{16x^{12}}} = \frac{3}{2|x^3|}$ $\boxed{x \neq 0}$ $\longrightarrow$

m) $\sqrt[3]{\frac{-4}{27x^6}} = \frac{\sqrt[3]{-4}}{3x^2}$ $\boxed{x \neq 0}$



7. $x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

a) $\sqrt[3]{\sqrt[4]{x^1}} = \left(x^{\frac{1}{4}}\right)^{\frac{1}{3}} = x^{\frac{1}{12}}$

$\downarrow$

$\sqrt[12]{x}$

e) $\sqrt{\sqrt{\sqrt{xy^2z^3}}} = \left(\left(\left(xy^2z^3\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = \sqrt[8]{xy^2z^3}$

bonus!

$\sqrt[4]{\sqrt[4]{\sqrt{\text{☺}^3}}}$ $\left(\left(\left(\text{☺}\right)^{\frac{3}{2}}\right)^{\frac{1}{4}}\right)^{\frac{1}{4}} = \sqrt[32]{\text{☺}^3}$

1.3 Simplifying Radicals

↳ variables represent non-negative real #'s

#3.

a) $\sqrt{32} = 4\sqrt{2}$

c) $\sqrt{75} = 5\sqrt{3}$

e) $\sqrt[3]{16} = 2\sqrt[3]{2}$

g) $3\sqrt{45} = 9\sqrt{5}$

i) $\frac{4}{3}\sqrt{54} = 4\sqrt{6}$

k) $\frac{5}{6}\sqrt[3]{-32} = -\frac{5}{3}\sqrt[3]{4}$

m) $\sqrt[4]{162} = 3\sqrt[4]{2}$

$\cancel{\frac{4}{3}\sqrt{9}}\cdot\sqrt{6}$

$\frac{5}{6}\sqrt[3]{-8}\cdot\sqrt[3]{4}$

$\frac{5}{\cancel{6}\,3}(-\cancel{2})\sqrt[3]{4} = -\frac{5}{3}\sqrt[3]{4}$

#4.

a) $\sqrt{x^3y^2} = xy\sqrt{x}$

c) $\sqrt{18x^6} = 3x^3\sqrt{2}$

e) $\sqrt{\frac{5}{x^4}} = \frac{\sqrt{5}}{x^2}$

g) $\sqrt[3]{40x^3} = 2\sqrt[3]{5}\,x$

i) $\sqrt[3]{-64x^3y^6} = -4xy^2$

k) $\sqrt[4]{16x^4y^8} = 2x\,y^2$

m) $\sqrt[5]{-32x^4} = -2\sqrt[5]{x^4}$

#5. Write as an entire radical.

a) $2\sqrt{3} = \sqrt{2^2 \times 3} = \sqrt{12}$

g) $3\sqrt[3]{4} = \sqrt[3]{3^3 \times 4} = \sqrt[3]{27 \times 4} = \sqrt[3]{108}$

m) $2\sqrt[5]{3} = \sqrt[5]{2^5 \times 3} = \sqrt[5]{32 \times 3} = \sqrt[5]{96}$

p.19 #7

a) $x\sqrt{y} = \sqrt{yx^2}$

c) $2x\sqrt[3]{5xy} = \sqrt[3]{40x^4y}$

e) $-xy\sqrt[4]{xy^3} = -\sqrt[4]{x^5y^7}$

g) $\dfrac{x^2\sqrt[4]{xy}}{y} = \sqrt[4]{\dfrac{x^8 \cdot xy}{y^4}} = \sqrt[4]{\dfrac{x^9}{y^3}}$

i) $\dfrac{\sqrt[3]{20x^2}}{4x} = \sqrt[3]{\dfrac{20x^2}{4^3x^3}} = \sqrt[3]{\dfrac{20x^2}{64x^3}} = \sqrt[3]{\dfrac{5}{16x}}$

#8. a) $(\sqrt{3a^2b})(\sqrt{6ab^5}) = \sqrt{18a^3b^6}$

$= \cancel{\sqrt{9}} \times \sqrt{2} \times \cancel{\sqrt{a^2}} \times \sqrt{a} \times \cancel{\sqrt{b^6}}$

$= 3ab^3\sqrt{2a}$

b) $(4x\sqrt{10xy})(3y\sqrt{2x}) =$

$24x^2y \quad \sqrt{5y}$

c) $(2x\sqrt[3]{2y^4})(x^2\sqrt[3]{4y^2}) = 2x^3\sqrt[3]{8y^6}$

$= 4x^3y^2$

f) $\dfrac{\sqrt[3]{81x^2y^5}}{\sqrt[3]{x^5y}} = \sqrt[3]{\dfrac{81x^2y^5}{x^5y^1}} = \sqrt[3]{\dfrac{81y^4}{x^3}} = \sqrt[3]{\dfrac{27\times 3\times y^3\times y}{x^3}}$

$= \dfrac{3y\sqrt[3]{3y}}{x}$

d) $(ab\sqrt[3]{2ab^2})(3a\sqrt[3]{4a^2b^2}) = 3a^2b\sqrt[3]{\cancel{8}\cancel{a^3}b^4}$

$= 6a^3b^2\sqrt[3]{b}$

$= 3a^2b\cdot 2ab\sqrt[3]{b}$

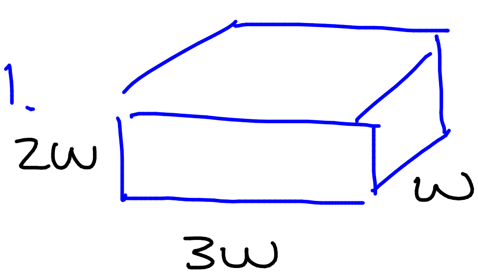
$= 6a^3b^2\sqrt[3]{b}$

e) $\dfrac{9x^2\sqrt{x^2y^5}}{3x^5\sqrt{x^6y^1}} = \dfrac{3}{x^3}\sqrt{\dfrac{y^4}{x^4}} = \dfrac{3y^2}{x^5}$

$\dfrac{3}{x^3}\sqrt{\dfrac{y^4}{x^4}} = \dfrac{3}{x^3}\cdot\dfrac{y^2}{x^2} = \dfrac{3y^2}{x^5}$

p. 20

\# 11.



$V = 192\text{ cm}^3$

height is twice width $h = 2w$

length is three times width $l = 3w$

Find dimensions.

$192 = w\cdot 3w\cdot 2w$

$192 = 6w^3$

$32 = w^3$

$w = \sqrt[3]{32}$

w: $2\sqrt[3]{4}$ cm

h: $4\sqrt[3]{4}$ cm

l: $6\sqrt[3]{4}$ cm

$w = 2\sqrt[3]{4}$

$h = 2w$

$l = 3w$

#14. What is the length of the diagonal?